Exercise 76

Show that 5 is a critical number of the function

$$g(x) = 2 + (x-5)^3$$

but g does not have a local extreme value at 5.

Solution

Recall that a critical number of a function $g(x)$ is a value of x such that its derivative is zero or nonexistent. Take the derivative of the function.

$$\begin{aligned} g'(x) &= \frac{d}{dx}[2 + (x-5)^3] \\ &= \frac{d}{dx}(2) + \frac{d}{dx}(x-5)^3 \\ &= 0 + 3(x-5)^2 \cdot \frac{d}{dx}(x-5) \\ &= 3(x-5)^2 \cdot (1) \\ &= 3(x-5)^2 \end{aligned}$$

Set $g'(x) = 0$ and solve for x.

$$3(x-5)^2 = 0$$

$$x - 5 = 0$$

$$x = 5$$

Therefore, 5 is a critical number of the function $g(x) = 2 + (x-5)^3$. g does not have an extreme value here, however, because there's no neighborhood about $x = 5$ for which $g(5)$ is a local minimum or a local maximum. For example,

$$\begin{cases} g(5.1) \approx 2.001 \\ g(5) = 2 \\ g(4.9) \approx 1.999 \end{cases} \quad \text{or} \quad \begin{cases} g(5.01) \approx 2.000001 \\ g(5) = 2 \\ g(4.99) \approx 1.999999 \end{cases}.$$

This is shown in the graph of $g(x)$ versus x. At $x = 5$ there's not an extreme value but rather an inflection point.